Class – XII
MATHEMATICS- 041
SAMPLE QUESTION PAPER 2019-20

Time: 3 Hrs. **Maximum Marks: 80**

General Instructions:

(i) All the questions are compulsory.
(ii) The question paper consists of 36 questions divided into 4 sections A, B, C, and D.
(iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.
(iv) There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of calculators is not permitted.

	SECTION A	
Q1 - Q10 are multiple choice type questions. Select the correct option		
1	If A is any square matrix of order 3 × 3 such that $\|A\| = 3$, then the value of $\|adjA\|$ is ? (a) 3 (b) $\frac{1}{3}$ (c) 9 (d) 27	1
2	Suppose P and Q are two different matrices of order 3 × n and n × p , then the order of the matrix P × Q is? (a) 3 × p (b) p × 3 (c) n × n (d) 3 × 3	1
3	If $\left(2\hat{i} + 6\hat{j} + 27\hat{k}\right) \times \left(\hat{i} + p\hat{j} + q\hat{k}\right) = \vec{0}$,then the values of p and q are ? (a) p= 6 ,q=27(b)p=3,q=$\frac{27}{2}$ (c) p=6,q= $\frac{27}{2}$ (d) p=3 ,q=27	1
4	If A and B are two events such that P(A)=0.2 , P(B)=0.4 and P(A ∪ B)=0.5 , then value of P(A/B) is ? (a)0.1 (b)0.25 (c)0.5 (d) 0.08	1
5	The point which does not lie in the half plane $2x + 3y - 12 \leq 0$ is (a) (1,2) (b) (2,1) (c) (2,3) (d)(−3,2)	1
6	If $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$, then the value of $\cos^{-1} x + \cos^{-1} y$ is ________ (a)$\frac{2\pi}{3}$(b)$\frac{\pi}{3}$(c)$\frac{\pi}{2}$(d) π	1

7	An urn contains 6 balls of which two are red and four are black. Two balls are drawn at random. Probability that they are of the different colours is (a) $\frac{2}{5}$(b) $\frac{1}{15}$ (c) $\frac{8}{15}$(d) $\frac{4}{15}$	1
8	$\int \frac{dx}{\sqrt{9-25x^2}}$ (a) $\sin^{-1}\left(\frac{5x}{3}\right) + c(b)\frac{1}{5}\sin^{-1}\left(\frac{5x}{3}\right) + c$ (c) $\frac{1}{6}\log\left(\frac{3+5x}{3-5x}\right) + c(d)\frac{1}{30}\log\left(\frac{3+5x}{3-5x}\right) + c$	1
9	What is the distance(in units) between the two planes $3x + 5y + 7z = 3$ and $9x + 15y + 21z = 9$? (a) 0(b) 3(c) $\frac{6}{\sqrt{83}}$(d) 6	1
10	The equation of the line in vector form passing through the point$(-1,3,5)$ and parallel to line $\frac{x-3}{2} = \frac{y-4}{3}$, $z = 2$. is (a) $\vec{r} = (-\hat{i} + 3\hat{j} + 5\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + \hat{k})$. (b) $\vec{r} = (-\hat{i} + 3\hat{j} + 5\hat{k}) + \lambda(2\hat{i} + 3\hat{j})$ (c) $\vec{r} = (2\hat{i} + 3\hat{j} - 2\hat{k}) + \lambda(-\hat{i} + 3\hat{j} + 5\hat{k})$ (d) $\vec{r} = (2\hat{i} + 3\hat{j}) + \lambda(-\hat{i} + 3\hat{j} + 5\hat{k})$	1
(Q11 - Q15) Fill in the blanks		
11	If f be the greatest integer function defined as$f(x) = [x]$ and g be the modulus function defined as $g(x) = \|x\|$, then the value of g of$\left(-\frac{5}{4}\right)$ is____________	1
12	If the function$f(x) = \begin{cases} \frac{x^2-1}{x-1} \text{ when } & x \neq 1 \\ k \text{ when } & x = 1 \end{cases}$ is given to be continuous at $x = 1$, then the value of k is _____	1
13	If$\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$**,** then value of y is _____**.**	1
14	If tangent to the curve $y^2 + 3x - 7 = 0$ at the point (h,k) is parallel to line $x - y = 4$, then value of k is ___? **OR** For the curve $y = 5x - 2x^3$,if x increases at the rate of 2units/sec, then at $x = 3$the slope of the curve is changing at_________	1
15	The magnitude of projection of $(2\hat{i} - \hat{j} + \hat{k})$ on $(\hat{i} - 2\hat{j} + 2\hat{k})$is_________ **OR** Vector of magnitude 5 units and in the direction opposite to $2\hat{i} + 3\hat{j} - 6\hat{k}$ is___	1
(Q16 - Q20) Answer the following questions		
16	Check whether (l + m + n) is a factor of the determinant$\begin{vmatrix} l+m & m+n & n+l \\ n & l & m \\ 2 & 2 & 2 \end{vmatrix}$or not. Give reason.	1
17	Evaluate $\int_{-2}^{2}(x^3 + 1)dx$.	1
18	Find $\int \frac{3+3\cos x}{x+\sin x} dx$. **3**	1

	OR Find $\int (cos^2 2x - sin^2 2x)dx$	
19	Find $\int xe^{(1+x^2)}dx$.	1
20	Write the general solution of differential equation $\frac{dy}{dx} = e^{x+y}$	1
	SECTION – B	
21	Express $\sin^{-1}\left(\frac{\sin x+\cos x}{\sqrt{2}}\right)$;where$-\frac{\pi}{4} < x < \frac{\pi}{4}$, in the simplest form. **OR** Let R be the relation in the set Z of integers given by R = {(a, b) : 2 divides a – b}.Show that the relation R transitive? Write the equivalence class [0].	2
22	If $= ae^{2x} + be^{-x}$, then show that $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$.	2
23	A particle moves along the curve $x^2 = 2y$. At what point, ordinate increases at the same rate as abscissa increases?	2
24	For three non-zero vectors $\vec{a}, \vec{b}$ and $\vec{c}$, prove that $[\vec{a} - \vec{b} \quad \vec{b} - \vec{c} \quad \vec{c} - \vec{a}] = 0$. **OR** If $\vec{a} + \vec{b} + \vec{c} = 0$ *and* $\lvert\vec{a}\rvert = 3, \lvert\vec{b}\rvert = 5, \lvert\vec{c}\rvert = 7$, then find the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$.	2
25	Find the acute angle between the lines $\frac{x-4}{3} = \frac{y+3}{4} = \frac{z+1}{5}$ and $\frac{x-1}{4} = \frac{y+1}{-3} = \frac{z+10}{5}$	2
26	A speaks truth in 80% cases and B speaks truth in 90%cases. In what percentage of cases are they likely to agree with each other in stating the same fact?	2
	SECTION – C	
27	Let f: A → B be a function defined as $f(x) = \frac{2x+3}{x-3}$, whereA = R – {3}and B = R – {2}. Is the function f one –one and onto? Is f invertible? If yes, then find its inverse.	4
28	If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, then prove that $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$. **OR** If $x = a(\cos 2\theta + 2\theta \sin 2\theta)$ and $y = a(\sin 2\theta - 2\theta \cos 2\theta)$, find$\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{8}$.	4
29	Solve the differential equation $x\,dy - y\,dx = \sqrt{x^2 + y^2}\,dx$.	4

30	Evaluate $\int_1^3 \lvert x^2 - 2x\rvert dx$.	4
31	Two numbers are selected at random (without replacement) from first 7 natural numbers. If X denotes the smaller of the two numbers obtained, find the probability distribution of X. Also, find mean of the distribution. **OR** There are three coins, one is a two headed coin (having head on both the faces), another is a biased coin that comes up heads 75% of the time and the third is an unbiased coin. One of the three coins is chosen at random and tossed. If It shows head. What is probability that it was the two headed coin ?	4
32	Two tailors A and B earn ₹150 and ₹200 per day respectively. A can stitch 6 shirts and 4 pants per day, while B can stitch 10 shirts and 4 pants per day. Form a L.P.P to minimize the labour cost to produce (stitch) at least 60 shirts and 32 pants and solve it graphically.	4
	SECTION D	
33	Using the properties of determinants, prove that $\begin{vmatrix} (y+z)^2 & x^2 & x^2 \\ y^2 & (z+x)^2 & y^2 \\ z^2 & z^2 & (x+y)^2 \end{vmatrix} = 2xyz(x+y+z)^3$. **OR** If A = $\begin{bmatrix} 2 & 3 & 4 \\ 1 & -1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$, find A^{-1}. Hence, solve the system of equations $x - y = 3$; $2x + 3y + 4z = 17$; $y + 2z = 7$	6
34	Using integration, find the area of the region $\{(x, y) : x^2 + y^2 \leq 1,\ x + y \geq 1,\ x \geq 0,\ y \geq 0\}$	6
35	A given quantity of metal is to be cast into a solid half circular cylinder with a rectangular base and semi-circular ends. Show that in order that total surface area is minimum, the ratio of length of cylinder to the diameter of semi-circular ends is $\pi : \pi + 2$. **OR** Show that the triangle of maximum area that can be inscribed in a given circle is an equilateral triangle.	6
36	Find the equation of a plane passing through the points $A(2,1,2)$ and $B(4,-2,1)$ and perpendicular to plane $\vec{r}.(\hat{i} - 2\hat{k}) = 5$. Also, find the coordinates of the point, where the line passing through the points (3,4,1) and (5,1,6) crosses the plane thus obtained.	6

Class – XII
MATHEMATICS (041)
SQP Marking Scheme (2019-20)

TIME: 3 Hrs. **Maximum Marks: 80**

	SECTION A	
1	(c) 9	1
2	(a) 3 × p	1
3	(b)p=3,q=$\frac{27}{2}$	1
4	(b)0.25	1
5	(c) (2,3)	1
6	(b)$\frac{\pi}{3}$	1
7	(c) $\frac{\mathbf{8}}{\mathbf{15}}$	1
8	(b) $\frac{1}{5}\sin^{-1}\left(\frac{5x}{3}\right) + c$	1
9	(a) 0	1
10	(b) $\vec{\boldsymbol{r}} = \left(-\hat{\boldsymbol{\imath}} + 3\hat{\boldsymbol{\jmath}} + 5\hat{\boldsymbol{k}}\right) + \boldsymbol{\lambda}(2\hat{\boldsymbol{\imath}} + 3\hat{\boldsymbol{\jmath}})$	1
11	$g\left(\left[-\frac{5}{4}\right]\right) = g(-2) = 2$	1
12	2	1
13	$y = 2$	1
14	$\frac{-3}{2}$ OR decreasing at rate of 72 units/sec.	1
15	2 units OR $\frac{5}{7}\left(-2\hat{\imath} - 3\hat{\jmath} + 6\hat{k}\right)$	1
16	Apply$R_1 \to R_1 + R_2 \begin{vmatrix} l+m+n & m+n+l & n+l+m \\ n & l & m \\ 2 & 2 & 2 \end{vmatrix}$ $= 2(l+m+n)\begin{vmatrix} 1 & 1 & 1 \\ n & l & m \\ 1 & 1 & 1 \end{vmatrix}$; yes (l + m + n) is a factor	1
17	$\int_{-2}^{2}(x^3 + 1)dx = \int_{-2}^{2}(x^3)dx + \int_{-2}^{2}1dx = I_1 + I_2$ $= 0 + [x]_{-2}^{2}$ (As I_1 is odd function) =2+2 = 4	1

18	Let $x + \sin x = t$ So $(1 + \cos x)dx = dt$ $I = 3\int \frac{dt}{t} = 3\log\|t\| + c = 3\log\|(x + \sin x)\| + c$ or directly by writing formula $$\int \frac{f'(x)}{f(x)} dx = \log\|f(x)\| + c$$ **OR** $\int \cos 4x\, dx = \frac{\sin 4x}{4} + c$	1
19	let $(1 + x^2) = t$ so $2xdx = dt$ $\Rightarrow I = \frac{1}{2}\int e^t dt = \frac{1}{2}e^t + c = \frac{1}{2}e^{(1+x^2)} + c$	1
20	$\frac{dy}{dx} = e^x e^y$ $\Rightarrow \frac{dy}{e^y} = e^x dx$ integrating both sides $\Rightarrow -e^{-y} + c = e^x$ $\Rightarrow \mathbf{e^x + e^{-y} = c}$	1
	SECTION B	
21	$= \sin^{-1}\left(\frac{\sin x}{\sqrt{2}} + \frac{\cos x}{\sqrt{2}}\right)$ if $-\frac{\pi}{4} < x < \frac{\pi}{4}$ $= \sin^{-1}\left(\sin x \cos\frac{\pi}{4} + \cos x \sin\frac{\pi}{4}\right)$ if $-\frac{\pi}{4} + \frac{\pi}{4} < x + \frac{\pi}{4} < \frac{\pi}{4} + \frac{\pi}{4}$ $= \sin^{-1}\left(\sin\left(x + \frac{\pi}{4}\right)\right)$ if $0 < \left(x + \frac{\pi}{4}\right) < \frac{\pi}{2}$ i.e. principal values $= \left(x + \frac{\pi}{4}\right)$ **OR** Let 2 divides$(a - b)$ and 2 divides $(b - c)$: where $a, b, c \in Z$ So 2 divides$[(a - b) + (b - c)]$ 2 divides $(a - c)$: Yes relation R is transitive $[0] = \{0, \pm 2, \pm 4, \pm 6, ...\}$	1 1 1 1
22	$y = ae^{2x} + be^{-x}$..................(1) $\frac{dy}{dx} = 2ae^{2x} - be^{-x}$ ....(2) $\frac{d^2y}{dx^2} = 4ae^{2x} + be^{-x}$ ....(3) putting values on LHS $= \frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y$ $= (4ae^{2x} + be^{-x}) - (2ae^{2x} - be^{-x}) - 2(ae^{2x} + be^{-x})$ $= 4ae^{2x} + be^{-x} - 2ae^{2x} + be^{-x} - 2ae^{2x} - 2be^{-x}$ $= 0$	1 1

23	$x^2 = 2y$ ………(1) $\Longrightarrow 2x\frac{dx}{dt} = 2\frac{dy}{dt}$ (given $\frac{dy}{dt} = \frac{dx}{dt}$) $\Longrightarrow 2x\frac{dx}{dt} = 2\frac{dx}{dt}$ $\Longrightarrow x = 1$ from (1) $y = \frac{1}{2}$ so point is $\left(1, \frac{1}{2}\right)$	1 1
24	$= (\vec{a} - \vec{b}).\{(\vec{b} - \vec{c}) \times (\vec{c} - \vec{a})\}$ $=(\vec{a} - \vec{b}).\{\vec{b} \times \vec{c} - \vec{b} \times \vec{a} - \vec{c} \times \vec{c} + \vec{c} \times \vec{a}\}$ $=(\vec{a} - \vec{b}).\{\vec{b} \times \vec{c} - \vec{b} \times \vec{a} + \vec{c} \times \vec{a}\}$ ……$(\vec{c} \times \vec{c} = 0)$ $= (\vec{a} - \vec{b}).\{\vec{b} \times \vec{c} + \vec{a} \times \vec{b} + \vec{c} \times \vec{a}\}$ $= \vec{a}.(\vec{b} \times \vec{c}) + \vec{a}.(\vec{a} \times \vec{b}) + \vec{a}.(\vec{c} \times \vec{a}) - \vec{b}.(\vec{b} \times \vec{c}) - \vec{b}.(\vec{a} \times \vec{b}) - \vec{b}.(\vec{c} \times \vec{a})$ $= \vec{a}.(\vec{b} \times \vec{c}) + 0 + 0 - 0 - 0 - \vec{b}.(\vec{c} \times \vec{a})$ $= \vec{a}.(\vec{b} \times \vec{c}) - \vec{b}.(\vec{c} \times \vec{a})$ $=0$ (STP remains same if vectors $\vec{a}, \vec{b}, \vec{c}$ are changed in cyclic order)	 1 1
	OR	
	$(\vec{a} + \vec{b} + \vec{c}).(\vec{a} + \vec{b} + \vec{c}) = 0$ $\Rightarrow \vec{a}.\vec{a} + \vec{a}.\vec{b} + \vec{a}.\vec{c} + \vec{b}.\vec{a} + \vec{b}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} + \vec{c}.\vec{b} + \vec{c}.\vec{c} = 0.$ $\Longrightarrow \lvert\vec{a}\rvert^2 + \lvert\vec{b}\rvert^2 + \lvert\vec{c}\rvert^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 0$ $\Longrightarrow 3^2 + 5^2 + 7^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 0$ $\Longrightarrow 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = -(9 + 25 + 49)$ $\Longrightarrow (\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = -\frac{83}{2}$	1 $\frac{1}{2}$ $\frac{1}{2}$
25	Vector in the direction of first line $\vec{b} = (3\hat{\imath} + 4\hat{\jmath} + 5\hat{k})$ Vector in the direction of second line $\vec{d} = (4\hat{\imath} - 3\hat{\jmath} + 5\hat{k})$ Angle θ between two lines is given by $\cos\theta = \frac{\vec{b}.\vec{d}}{\lvert\vec{b}\rvert\lvert\vec{d}\rvert}$ $\cos\theta = \frac{(3\hat{\imath} + 4\hat{\jmath} + 5\hat{k}).(4\hat{\imath} - 3\hat{\jmath} + 5\hat{k})}{\lvert(3\hat{\imath} + 4\hat{\jmath} + 5\hat{k})\rvert\lvert(4\hat{\imath} - 3\hat{\jmath} + 5\hat{k})\rvert}$ $\Longrightarrow \cos\theta = \frac{12 - 12 + 25}{\sqrt{9 + 16 + 25}\sqrt{9 + 16 + 25}}$ $\Longrightarrow \cos\theta = \frac{25}{\sqrt{50}\sqrt{50}}$ $\Longrightarrow \cos\theta = \frac{1}{2}$ $\Longrightarrow \theta = \frac{\pi}{3}$	 1 $\frac{1}{2}$ $\frac{1}{2}$
26	$P(A)=\frac{80}{100} = \frac{4}{5}$, $P(B)=\frac{90}{100} = \frac{9}{10}$ P(Agree)=P(Both speaking truth or both telling lie) $=P(AB \text{ or } \bar{A}\bar{B})$	 1

	$= P(A)P(B) or P(\bar{A})P(\bar{B})$ $=\left(\frac{4}{5}\right)\left(\frac{9}{10}\right)+\left(\frac{1}{5}\right)\left(\frac{1}{10}\right)$ $=\frac{36+1}{50}=\frac{37}{50}$ $=\frac{74}{100}$ = 74%	1
	SECTION C	
27	Let $y = f(x) = \frac{2x+3}{x-3}$ ……………………….(1) Let $x_1, x_2 \in A = R - \{3\}$ Let $f(x_1) = f(x_2)$ $\Rightarrow \frac{2x_1+3}{x_1-3} = \frac{2x_2+3}{x_2-3}$ $\Rightarrow (2x_1+3)(x_2-3) = (2x_2+3)(x_1-3)$ $\Rightarrow (2x_1x_2 - 6x_1 + 3x_2 - 9) = (2x_1x_2 - 6x_2 + 3x_1 - 9)$ $\Rightarrow -6x_1 + 3x_2 = -6x_2 + 3x_1$ $\Rightarrow 9x_1 = 9x_2$ $\Rightarrow x_1 = x_2$ Now $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ so $f(x)$ is one-one For onto $y = \frac{2x+3}{x-3}$ $\Rightarrow xy - 3y = 2x + 3$ $\Rightarrow xy - 2x = 3y + 3$ $\Rightarrow x(y-2) = 3(y+1)$ $\Rightarrow x = \frac{3(y+1)}{(y-2)}$ ……………………….(2) equation (2) is defined for all real values of y except 2 i.e $y \in R - \{2\}$ which is same as given set $B = R - \{2\}$ (co-domain=range) Also $y = f(x)$ $f(x) = f\left(\frac{3(y+1)}{(y-2)}\right)$ $= \frac{2\left[\frac{3(y+1)}{(y-2)}\right]+3}{\frac{3(y+1)}{(y-2)}-3}\left(\text{since } f(x) = \frac{2x+3}{x-3}\right)$ $\frac{2(3y+3)+3(y-2)}{3y+3-3y+6} = \frac{9y}{9} = y$ Thus for every y ∈ B,there exists x ∈ A such that $f(x) = y$ Thus function is onto. Since f(x) is one-one and onto so f(x)is invertible. Inverse is given by $x = f^{-1}(y) = \frac{3(y+1)}{(y-2)}$	$\frac{1}{2}$ 1 $1\frac{1}{2}$ 1
28	$\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$ Let $x = \sin A$, $y = \sin B$ $\sqrt{1-\sin^2 A} + \sqrt{1-\sin^2 B} = a(\sin A - \sin B)$ $\cos A + \cos B = a(\sin A - \sin B)$ $\Longrightarrow 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) = 2a\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$ $\Longrightarrow \cos\left(\frac{A-B}{2}\right) = a\sin\left(\frac{A-B}{2}\right)$	$\frac{1}{2}$ 1

	$\Rightarrow \cot\left(\frac{A-B}{2}\right) = a$ $\Rightarrow \frac{A-B}{2} = \cot^{-1} a$ $\Rightarrow A - B = 2\cot^{-1} a$ $\Rightarrow \sin^{-1} x - \sin^{-1} y = 2\cot^{-1} a$ differentiating w.r.t. x $\Rightarrow \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0$ $\Rightarrow \frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$ **OR**	1 1 $\frac{1}{2}$	
	$x = a(\cos 2\theta + 2\theta \sin 2\theta)$ $\Rightarrow \frac{dx}{d\theta} = a(-2\sin 2\theta + 2\sin 2\theta + 4\theta\cos 2\theta)$ $\Rightarrow \frac{dx}{d\theta} = a(4\theta\cos 2\theta)$……………..(1) $y = a(\sin 2\theta - 2\theta\cos 2\theta)$ $\Rightarrow \frac{dy}{d\theta} = a(2\cos 2\theta + 4\theta\sin 2\theta - 2\cos 2\theta)$ $\Rightarrow \frac{dy}{d\theta} = a(4\theta\sin 2\theta)$………………(2) using (1)and (2) $\Rightarrow \frac{dy}{dx} = \frac{a(4\theta\sin 2\theta)}{a(4\theta\cos 2\theta)}$ $\Rightarrow \frac{dy}{dx} = \frac{\sin 2\theta}{\cos 2\theta} = \tan 2\theta$ Differentiating again with respect to x, we get $\Rightarrow \frac{d^2y}{dx^2} = 2\,sec^2 2\theta.\frac{d\theta}{dx}$ $\Rightarrow \frac{d^2y}{dx^2} = 2\,sec^2 2\theta.\frac{1}{a(4\theta\cos 2\theta)}$ $\left.\frac{d^2y}{dx^2}\right]_{\theta=\frac{\pi}{8}} = 2\,sec^2\frac{\pi}{4}.\frac{1}{a\left(4\frac{\pi}{8}\cos\frac{\pi}{4}\right)}$ $= \frac{8\sqrt{2}}{\pi a}$	1 1 $\frac{1}{2}$ $\frac{1}{2}$ 1	
29	$x\frac{dy}{dx} - y = \sqrt{x^2+y^2}$ $\Rightarrow x\frac{dy}{dx} = y + \sqrt{x^2+y^2}$ $\Rightarrow \frac{dy}{dx} = \frac{y+\sqrt{x^2+y^2}}{x}$…………………………………(1) let y = vx differentiating with w.r.t. x $\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ put in (1)	1	

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$$\Rightarrow v + x\frac{dv}{dx} = \frac{vx + \sqrt{x^2 + v^2x^2}}{x}$$
$$\Rightarrow v + x\frac{dv}{dx} = \frac{x(v + \sqrt{1 + v^2})}{x}$$
$$\Rightarrow x\frac{dv}{dx} = v + \sqrt{1 + v^2} - v$$
$$\Rightarrow x\frac{dv}{dx} = \sqrt{1 + v^2}$$
$$\Rightarrow \frac{dv}{\sqrt{1 + v^2}} = \frac{dx}{x}$$
integrating both sides
$$\Rightarrow \int \frac{dv}{\sqrt{1 + v^2}} = \int \frac{dx}{x}$$
$$\Rightarrow \log\left(v + \sqrt{1 + v^2}\right) = \log x + \log c$$
$$\Rightarrow \log\left(v + \sqrt{1 + v^2}\right) = \log cx$$
$$\Rightarrow \left(v + \sqrt{1 + v^2}\right) = cx$$
$$\Rightarrow \left(\frac{y}{x} + \sqrt{1 + \left(\frac{y}{x}\right)^2}\right) = cx$$
$$\Rightarrow y + \sqrt{x^2 + y^2} = cx^2$$
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Consider $I=\int_1^3 |x^2 - 2x| dx$
$$|x^2 - 2x| = \begin{cases} -(x^2 - 2x) & when\ 1 \le x < 2 \\ (x^2 - 2x) & when\ 2 \le x \le 3 \end{cases}$$
$I=\int_1^2 |x^2 - 2x| dx + \int_2^3 |x^2 - 2x| dx$
$I=\int_1^2 -(x^2 - 2x)\,dx + \int_2^3 (x^2 - 2x) dx$
$I=-\left[\frac{x^3}{3} - x^2\right]_1^2 + \left[\frac{x^3}{3} - x^2\right]_2^3$
$I=-\left(-\frac{4}{3} + \frac{2}{3}\right) + \left(\frac{4}{3}\right)$
$I=\frac{6}{3} = 2$
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Let X denotes the smaller of the two numbers obtained
So X can take values 1,2,3,4,5,6
P(X=1 is smaller number)
P(X=1)=$\frac{6}{7_{C_2}} = \frac{6}{21} = \frac{2}{7}$
(Total cases when two numbers can be selected from first 7 numbers are 7_{C_2})
P(X=2)=$\frac{5}{7_{C_2}} = \frac{5}{21}$
P(X=3)=$\frac{4}{7_{C_2}} = \frac{4}{21}$
P(X=4)=$\frac{3}{7_{C_2}} = \frac{3}{21} = \frac{1}{7}$
P(X=5)=$\frac{2}{7_{C_2}} = \frac{2}{21}$
P(X=6)=$\frac{1}{7_{C_2}} = \frac{1}{21}$
<table>
<tr><td>x_i</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
</table>
</td>
<td>$\frac{1}{2}$

2</td>
</tr>
</table>

p_i	$\frac{6}{21}$	$\frac{5}{21}$	$\frac{4}{21}$	$\frac{3}{21}$	$\frac{2}{21}$	$\frac{1}{21}$
$p_i x_i$	$\frac{6}{21}$	$\frac{10}{21}$	$\frac{12}{21}$	$\frac{12}{21}$	$\frac{10}{21}$	$\frac{6}{21}$

$\frac{1}{2}$

Mean $=\sum p_i x_i = \frac{6}{21} + \frac{10}{21} + \frac{12}{21} + \frac{12}{21} + \frac{10}{21} + \frac{6}{21} = \frac{56}{21} = \frac{8}{3}$ — 1

OR

Let E_1 = event of selecting a two headed coin

E_1 = event of selecting a biased coin, which shows 75% times Head

E_3 =event of selecting a unbiased coin.

A = event that tossed coin shows head. — 1

$$\therefore P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$$ — $\frac{1}{2}$

. $P\left(A/E_1\right) = P$(coin showing head given that it is two headed coin)

$= 1$

$P\left(A/E_2\right) = P$(coin showing head given that it is a biased coin)

$= \frac{75}{100} = \frac{3}{4}$

$P\left(A/E_3\right) = P$(coin showing head given that it is unbiased coin) — 1

$= \frac{1}{2}$

By Bayes theorem

P(gettingtwo headedcoin when it is known that it shows Head)

$$P\left(E_1/A\right) = \frac{P(E_1)P\left(A/E_1\right)}{P(E_1)P\left(A/E_1\right) + P(E_2)P\left(A/E_2\right) + P(E_1)P\left(A/E_3\right)}$$

$$= \frac{\frac{1}{3}\times 1}{\frac{1}{3}\times 1 + \frac{1}{3}\times\frac{3}{4} + \frac{1}{3}\times\frac{1}{2}} = \frac{\frac{1}{3}\times 1}{\frac{1}{3}\left(1+\frac{3}{4}+\frac{1}{2}\right)} = \frac{\frac{1}{3}}{\frac{1}{3}\times\frac{9}{4}} = \frac{4}{9}$$ — $1\frac{1}{2}$

Required probability$=\frac{4}{9}$

32 Let tailor A works for x days and tailor B works for y days — $\frac{1}{2}$

Objective function :

To minimize labour cost $Z = 150x + 200y$ (in ₹)

Subject to constraints

$6x + 10y \geq 60$ i.e. $3x + 5y \geq 30$

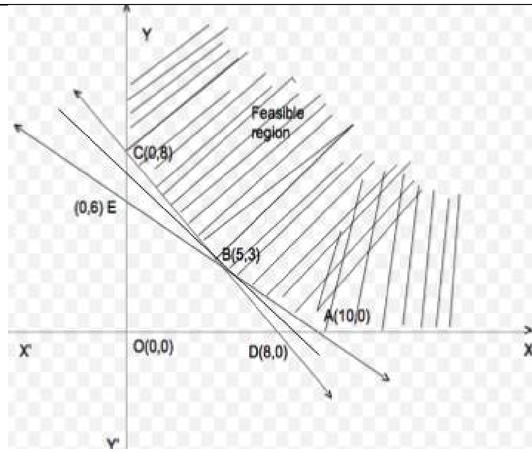
$4x + 4y \geq 32$ i.e. $x + y \geq 8$ — $1\frac{1}{2}$

$x \geq 0 , y \geq 0$

consider equations to draw the graph and then we will shade feasible region

$$3x + 5y = 30$$

$$x + y = 8$$



1

corner points of feasible region are A(10,0),B(5,3) and C(0,8)
Value of Z at these corner points

Point	$Z = 150x + 200y$ (in ₹)
A(10,0)	=1500+0=1500
B(5,3)	=750+600=1350 (minimum)
C(0,8)	=0+1600=1600

So minimum value of Z is ₹1350 when tailor A works for 5 days and tailor B works for 3 days.
To check draw $150x + 200y < 1350$ i.e $3x + 4y < 27$
As there is no region common with feasible region so minimum value is ₹1350 1

SECTION D

33

LHS= $\begin{vmatrix} (y+z)^2 & x^2 & x^2 \\ y^2 & (z+x)^2 & y^2 \\ z^2 & z^2 & (x+y)^2 \end{vmatrix}$

$Apply\ C_2 \to C_2 - C_1, \quad C_3 \to C_3 - C_1$

$= \begin{vmatrix} (y+z)^2 & x^2-(y+z)^2 & x^2-(y+z)^2 \\ y^2 & (z+x)^2-y^2 & 0 \\ z^2 & 0 & (x+y)^2-z^2 \end{vmatrix}$ 1

$= \begin{vmatrix} (y+z)^2 & (x+y+z)(x-y-z) & (x+y+z)(x-y-z) \\ y^2 & (z+x+y)(z+x-y) & 0 \\ z^2 & 0 & (x+y+z)(x+y-z) \end{vmatrix}$ 1

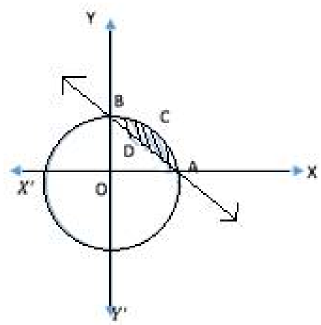
Taking $(x+y+z)$common from C_2as well as C_3

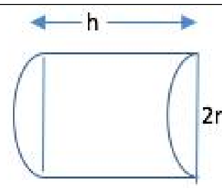
$=(x+y+z)^2 \begin{vmatrix} (y+z)^2 & (x-y-z) & (x-y-z) \\ y^2 & (z+x-y) & 0 \\ z^2 & 0 & (x+y-z) \end{vmatrix}$ 1

Apply $R_1 \to R_1 - R_2 - R_3$

$=(x+y+z)^2 \begin{vmatrix} 2yz & -2z & -2y \\ y^2 & (z+x-y) & 0 \\ z^2 & 0 & (x+y-z) \end{vmatrix}$

	Apply $C_2 \to y\,C_2$ and $C_3 \to z\,C_3$ $=\frac{(x+y+z)^2}{yz}\begin{vmatrix} 2yz & -2yz & -2yz \\ y^2 & (yz+yx-y^2) & 0 \\ z^2 & 0 & (zx+zy-z^2) \end{vmatrix}$ Apply $C_2 \to C_2 + C_1$ and $C_3 \to C_3 + C_1$ $=\frac{(x+y+z)^2}{yz}\begin{vmatrix} 2yz & 0 & 0 \\ y^2 & (yz+yx) & y^2 \\ z^2 & z^2 & (zx+zy) \end{vmatrix}$ expanding along R_1 $=\left(\frac{(x+y+z)^2}{yz}\right)2yz[(yz+yx)(zx+zy)-y^2z^2]$ $=2(x+y+z)^2[xyz^2+x^2yz+xy^2z+y^2z^2-y^2z^2]$ $=2xyz(x+y+z)^2(x+y+z)$ $=2xyz(x+y+z)^3$ **OR**	1 1 1
	** $A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & -1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$ $\lvert A\rvert = 2(-2)-3(2-0)+4(1-0) = -6 \neq 0$ $\therefore A^{-1}$ exists Cofactors $A_{11} = -2 \quad A_{12} = -2 \quad A_{13} = 1$ $A_{21} - 2 \quad A_{22} = 4 \quad A_{23} = -2$ $A_{31} = 4 \quad A_{32} = 4 \quad A_{33} = -5$ $Adj\ A = \begin{bmatrix} -2 & -2 & 1 \\ -2 & 4 & -2 \\ 4 & 4 & -5 \end{bmatrix}'$ $Adj\ A = \begin{bmatrix} -2 & -2 & 4 \\ -2 & 4 & 4 \\ 1 & -2 & -5 \end{bmatrix}$ $A^{-1} = \frac{Adj\ A}{\lvert A\rvert} = \frac{1}{-6}\begin{bmatrix} -2 & -2 & 4 \\ -2 & 4 & 4 \\ 1 & -2 & -5 \end{bmatrix}$ System of equations can be written as $AX = B$ Where $A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & -1 & 0 \\ 0 & 1 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 17 \\ 3 \\ 7 \end{bmatrix}$ Now $AX = B$ $\Rightarrow X = A^{-1}B$ $\Rightarrow X = \frac{1}{-6}\begin{bmatrix} -2 & -2 & 4 \\ -2 & 4 & 4 \\ 1 & -2 & -5 \end{bmatrix}\begin{bmatrix} 17 \\ 3 \\ 7 \end{bmatrix}$	1 2 1 $\frac{1}{2}$

	$$\Rightarrow X = \frac{1}{-6}\begin{bmatrix} -34-6+28 \\ -34+12+28 \\ 17-6-35 \end{bmatrix}$$ $$\Rightarrow X = \frac{1}{-6}\begin{bmatrix} -12 \\ 6 \\ -24 \end{bmatrix}$$ $$\Rightarrow X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$ $$\Rightarrow x = 2, \quad y = -1, \quad z = 4$$	$1\frac{1}{2}$
34	$x^2 + y^2 = 1$………………..(1) $x + y = 1$…………………..(2) solving (1) and(2) $x^2 + (1-x)^2 = 1$ $x^2 + x^2 - 2x + 1 = 1$ $2x^2 - 2x = 0$ $2x(x-1) = 0$ $x = 0$ or $x = 1$  Required area = shaded area ACBDA =area(OACBO)− area(OADBO) $=\int_0^1 (y_{circle} - y_{line})dx$ $\int_0^1 \sqrt{1-x^2}dx - \int_0^1 (1-x)dx$ $=\left[\frac{x\sqrt{1-x^2}}{2} + \frac{1}{2}\sin^{-1}x\right]_0^1 - \left[x - \frac{x^2}{2}\right]_0^1$ $\left[\left(0 + \frac{1}{2}.\frac{\pi}{2}\right) - 0\right] - \left[\left(1 - \frac{1}{2}\right)\right]$ $\left(\frac{\pi}{4} - \frac{1}{2}\right)$ square units	1 1 1 $1\frac{1}{2}$ $1\frac{1}{2}$
35	Let r be the radius and $h$be the height of half cylinder Volume $=\frac{1}{2}\pi r^2 h = V$(constant)……………..(1)	$\frac{1}{2}(fig)$



$1\frac{1}{2}$

Total surface area of half cylinder is

$S=2\left(\frac{1}{2}\pi r^2\right) + \pi rh + 2rh$…………………(2)

From (1) put the value of h in (2) 1

$S=(\pi r^2) + \pi r\left(\frac{2V}{\pi r^2}\right) + 2r\left(\frac{2V}{\pi r^2}\right)$

$S=(\pi r^2) + \left(\frac{1}{r}\right)\left[\frac{4V}{\pi} + 2V\right]$

$\frac{ds}{dr} = (2\pi r) + \left(\frac{-1}{r^2}\right)\left[\frac{4V}{\pi} + 2V\right]$ ……………..(3)

For maxima/minima $\frac{ds}{dr} = 0$

$\Rightarrow (2\pi r) + \left(\frac{-1}{r^2}\right)\left[\frac{4V}{\pi} + 2V\right] = 0$ 1

$\Rightarrow (2\pi r) = \left(\frac{1}{r^2}\right)\left[\frac{4V + 2V\pi}{\pi}\right]$

$\Rightarrow \pi r^3 = V\left[\frac{2+\pi}{\pi}\right]$ 1

$\Rightarrow V=\frac{\pi^2 r^3}{\pi+2}$………………..(4)

From (1) and (4)

$\Rightarrow \frac{1}{2}\pi r^2 h = \frac{\pi^2 r^3}{\pi + 2}$ 1

$\Rightarrow \frac{h}{2r} = \frac{\pi}{\pi + 2}$

$\Rightarrow height{:}\,diameter = \pi{:}\pi + 2$

Differentiating (3) with respect to r

$\frac{d^2s}{dr^2} = (2\pi) + \left(\frac{2}{r^3}\right)\left[\frac{4V}{\pi} + 2V\right]$=positive (as all quantities are +ve)

so S is minimum when

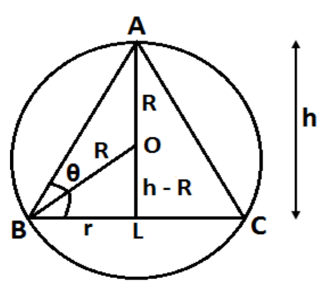
$height{:}\,diameter = \pi{:}\pi + 2$

OR

Let 2r be the base and h be the height of triangle ,which is inscribed in a circle of radius R

Area of triangle=$\frac{1}{2}(base)(height)$ 1

$A = \frac{1}{2}(2r)(h) = rh$…………(1)



$\frac{1}{2}(fig)$

Area being positive quantity, A will be maximum or minimum if A^2 is

	maximum or minimum. $Z = A^2 = r^2h^2$……………………..(2) Now In triangle OLB $BL^2 = OB^2 - OL^2$ In ΔOBD $Z = A^2 = r^2 h^2$ $r^2 = R^2 - (h-R)^2 \Rightarrow r^2 = 2hR - h^2$ $put\ in$ (2) $Z = h^2(2hR - h^2)$ $\Rightarrow Z = (2h^3R - h^4)$ $\Rightarrow \frac{dZ}{dh} = 6h^2R - 4h^3$……………(3) For maxima/minima $\frac{dZ}{dh} = 0$ $\Rightarrow 6h^2R - 4h^3 = 0$ $\Rightarrow 6R = 4h(h \neq 0)$ $\Rightarrow h = \frac{3R}{2}$ differentiating (3) w.r.t. h $\Rightarrow \frac{d^2Z}{dh^2} = 12hR - 12h^2$ $\Rightarrow \left.\frac{d^2Z}{dh^2}\right]_{h=\frac{3R}{2}} = 12\left(\frac{3R}{2}\right)R - 12\left(\frac{3R}{2}\right)^2$ $= 18R^2 - 27R^2 = -ve$ so Z=A^2 is maximum when $h = \frac{3R}{2}$ $\Rightarrow A$ is maximum when $h = \frac{3R}{2}$ when $h = \frac{3R}{2}$, $r^2 = 2hR - h^2 = 2R.\frac{3R}{2} - \left(\frac{3R}{2}\right)^2$ $r^2 = \frac{3R^2}{4}$ $r = \frac{\sqrt{3}R}{2}$ $\tan\theta = \frac{h}{r} = \frac{\frac{3R}{2}}{\frac{\sqrt{3}R}{2}} = \sqrt{3}\theta = \frac{\pi}{3}$ Triangle ABC is equilateral triangle	$\frac{1}{2}$ 1 1 1 1
36	Let $P(x,y,z)$ be any point on the plane in which $A(2,1,2)$ and $B(4,-2,1)$lie. $\therefore \overrightarrow{AP}\ and\ \overrightarrow{AB}$lie on required plane. Also required plane is perpendicular to given plane $\vec{r}.(\hat{i} - 2\hat{k}) = 5$ ∴normal to given plane $\overrightarrow{n_1} = (\hat{i} - 2\hat{k})$ lie on required plane. $\Rightarrow \overrightarrow{AP}, \overrightarrow{AB}and\overrightarrow{n_1}$ are coplanar. Where$\overrightarrow{AP} = (x-2)\hat{i} + (y-1)\hat{j} + (z-2)\hat{k}$ $\overrightarrow{AB}$ = =$2\hat{i} - 3\hat{j} - \hat{k}$ ⇒Scaler triple product $[\overrightarrow{AP}\ \ \overrightarrow{AB}\ \ \overrightarrow{n_1}] = 0$	1 1 1

	$\Rightarrow \begin{vmatrix} x-2 & y-1 & z-2 \\ 2 & -3 & -1 \\ 1 & 0 & -2 \end{vmatrix} = 0$ $\Rightarrow (x-2)(6-0) - (y-1)(-4+1) + (z-2)(0+3) = 0$ $\Rightarrow 6x - 12 + 3y - 3 + 3z - 6 = 0$ $\Rightarrow 2x + y + z = 7$…………………..(1) Line passing through points $L(3,4,1)$and $M(5,1,6)$ is $\Rightarrow \frac{x-3}{2} = \frac{y-4}{-3} = \frac{z-1}{5} = \lambda$………….(2) ⇒General point on the line is $Q(2\lambda+3, -3\lambda+4, 5\lambda+1)$ As line (2) crosses plane (1) so point Q should satisfy equation(1) $\therefore 2(2\lambda+3) + (-3\lambda+4) + (5\lambda+1) = 7$ $4\lambda + 6 - 3\lambda + 4 + 5\lambda + 1 = 7$ $6\lambda = -4$ $\lambda = -\frac{2}{3}$ $Q(-\frac{4}{3}+3, 2+4, -\frac{10}{3}+1) = Q\left(\frac{5}{3}, 6, -\frac{7}{3}\right)$	1 1 1